PreCalculus Notes – Sections 1.2, 1.5, 1.8, 1.6, and 1.7

Linear Functions

If you are given $f(-3)=5$, then

a) what is the input?

b) what is the output?

c) what POINT do you know is on the graph of $f(x)$?

If $f(-3)=5$ & $f(1)=-11$ answer the following questions:

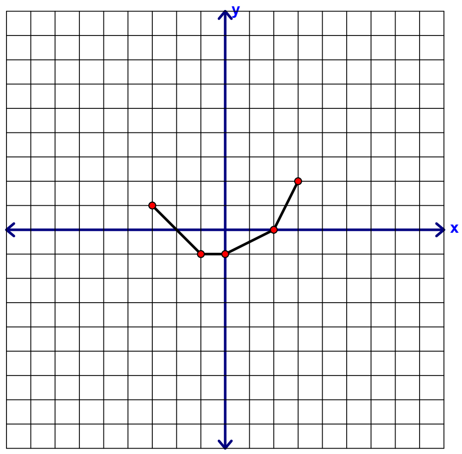
1) What is the linear function that contains the two values?

2) What is the distance between the two points?

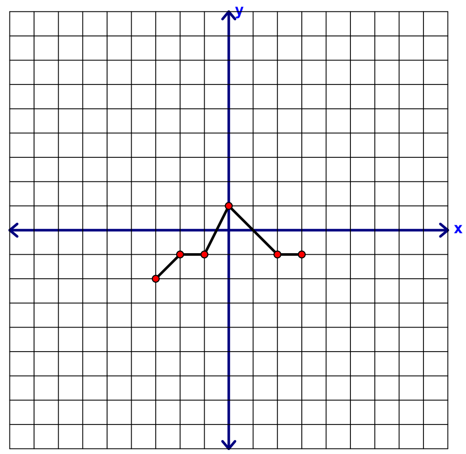
Using graphs to explore operations with functions

Use the graphs of f(x) and g(x) below to answer the questions:

f(x)



g(x)



Calculate: $(f)(0)$:

Calculate: $(g)(1)$:

Calculate: $(f-g)(-3)$:

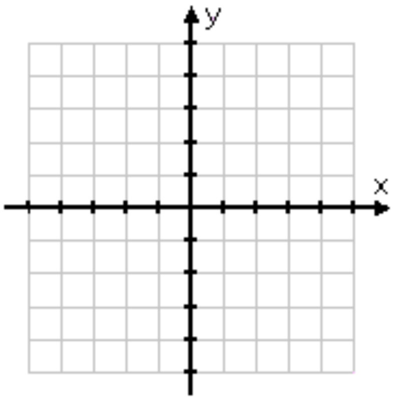
Calculate: $(f/g)(2)$:

Calculate: $(f \circ g)(3)$:

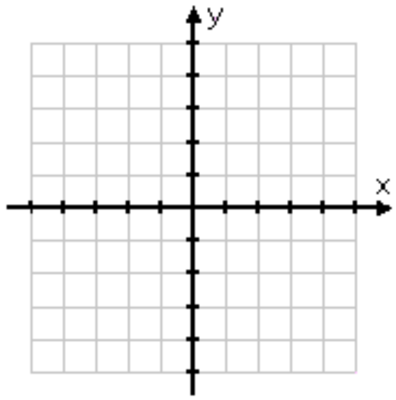
Calculate: $f\left(f\left(-2\right)\right)$:

Symmetry and Even/Odd Functions

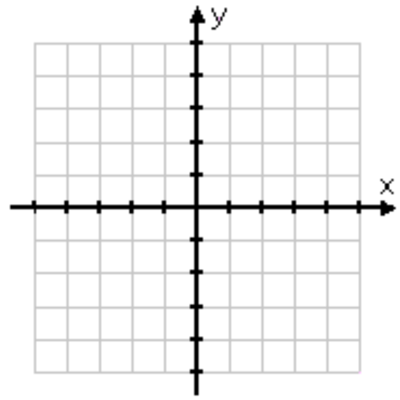
There are three different types of graph symmetry:



y-axis symmetry
EVEN function



origin symmetry
ODD function



x-axis symmetry
but
NEITHER even nor odd

- A function is **even** if the function is symmetric about the y-axis;
  this means $f(-x) = f(x)$
- A function is **odd** if the function is symmetric about the origin;
  this means $f(-x) = -f(x)$
- To find out whether the function is even or odd, substitute "-x" for "x" and simplify the function.
  - If it is the SAME as the original, it is even.
  - If it is the OPPOSITE of the original, it is odd

Decide, algebraically, if the following functions are even, odd or neither:

a) $y = x^4 - x^2 + 3$

b) $h(x) = x^5 + 1$

c) $g(x) = |x| - 2$

d) $g(x) = x^3 - x$

Standard Form of the Equation of a Circle

The point (x,y) lies on the circle of radius r and center (h,k) if and only if;

$$(x-h)^2+(y-k)^2=r^2$$

$\boxed{\text{Ex. 1:}}$ $x^2+y^2=16$ is a circle with its center at the _______ and radius of _______

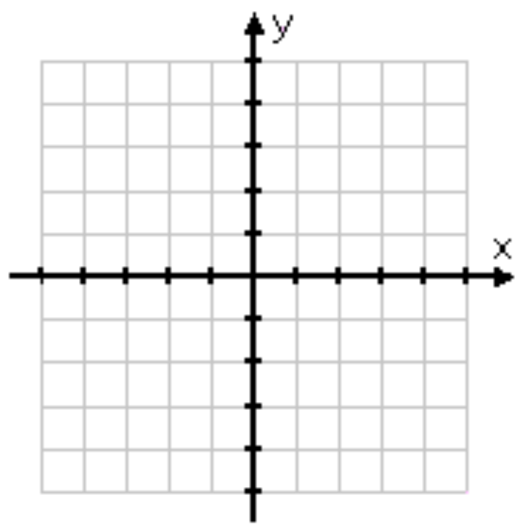
$\boxed{\text{Ex. 2:}}$ If h = 3, k = -2 and r = 4, give the equation of the circle:

$\boxed{\text{Ex. 3:}}$ $(x-2)^2+(y+1)^2=4$ is a circle

with its center at _______

and radius of ______________

Now sketch a graph!



Using your graph, determine

What is the domain of $(x-2)^2+(y+1)^2=4$?

What is the range of $(x-2)^2+(y+1)^2=4$?

Ex. 4: The point (3,4) lies on a circle whose center is at (-1,2).
Write the standard form of the equation of this circle:

Ex. 5: If the diameter of a circle has two endpoints of (-4,-1) and (4,1), write the equation of the circle:

Side skill: Completing the Square

Ex. 6: Use completing the square to change the look of these quadratic functions:

$F(x) = x^2 + 4x - 8$

$G(x) = x^2 - 10x - 12$

Why would I mention this now? For solving a problem involving circles like the following:

Ex. 7: Change this equation of a circle to be in standard form.

$x^2 + 2x - 5 + y^2 - 6y + 10 = 11$

Now identify the center and radius:

Selected homework from sections 1.2 and 1.5 and 1.8:

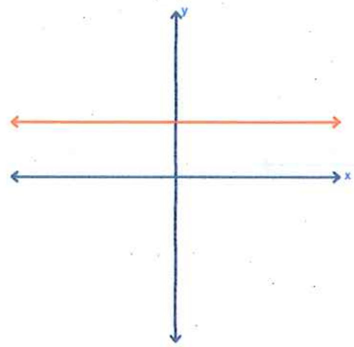
p. 22-24 # 17 – 27 odd, 59, 61, 63, 67, 69
p. 61-63 # 1,3, 13, 15, 19, 23, 71, 73, 75
p. 89 # 1, 3, 43, 45

Common "Parent" Functions

The eight graphs shown in the figures below represent the most commonly used functions in algebra. Familiarity with the basic characteristics of these simple graphs will help you analyze the shapes of more complicated graphs – in particular, graphs obtained from these common graphs by the rigid and non-rigid transformations that are studied in the next section.

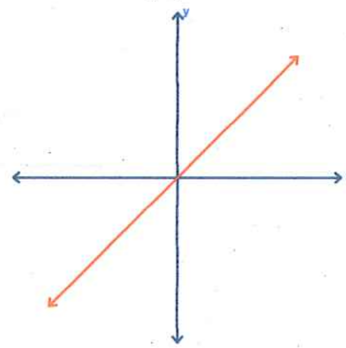
Constant

$f(x)=c$



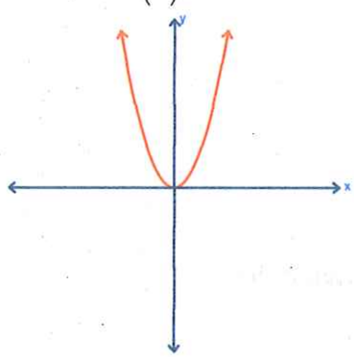
Identity (or Linear)

$f(x)=x$



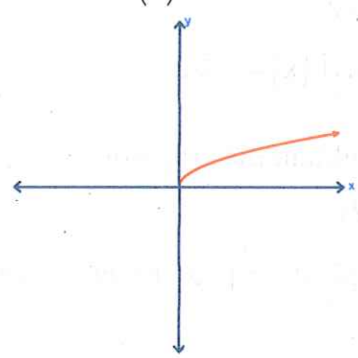
Quadratic

$f(x)=x^2$



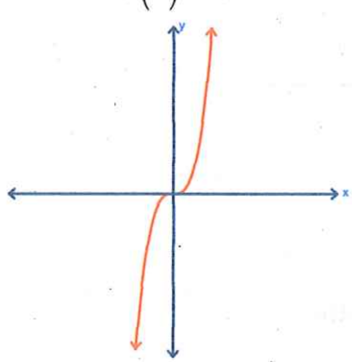
Square Root

$f(x)=\sqrt{x}$



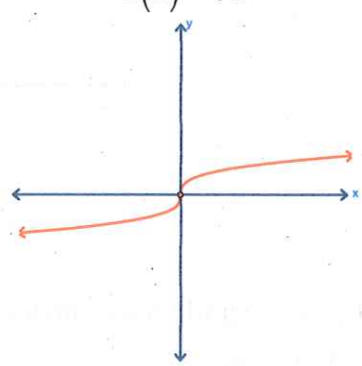
Cubic

$f(x)=x^3$



Cubic Root

$f(x)=\sqrt[3]{x}$

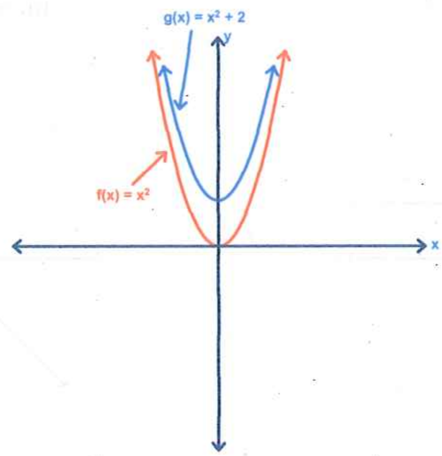


Shifting Graphs

Many functions have graphs that are simple transformations of the parent graphs we talked about in the previous section. For example, you can obtain the graph of

$$h(x) = x^2 + 2$$

by shifting the graph of $f(x) = x^2$ *upward* two units, as show below:



In function notation, *h* and *f* are related as follows:

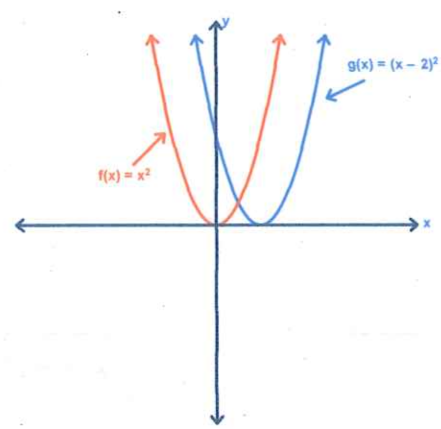
$$h(x) = f(x) + 2$$

which means $h(x) = x^2 + 2$

Similarly, you can obtain the graph of

$$g(x) = (x-2)^2$$

by shifting the graph of $f(x) = x^2$ *to the right* two units, as shown below:



In this case, the functions *g* and *f* have the following relationship

$$g(x) = f(x-2) = (x-2)^2$$

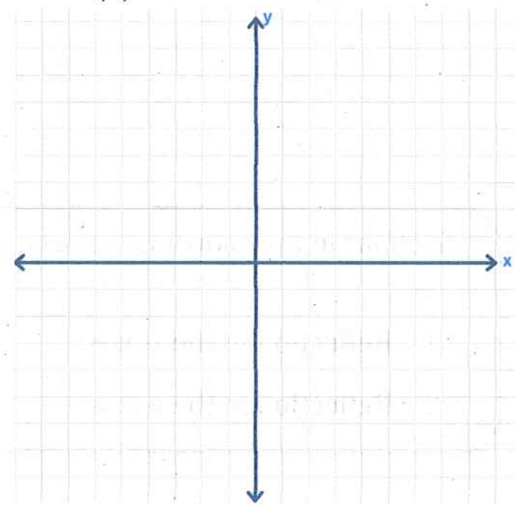
These shifts are called *rigid transformations* and can be summarized as shown below:

Let c be a positive real number. Vertical and horizontal shifts in the graph of $y=f(x)$, are represented as follows:

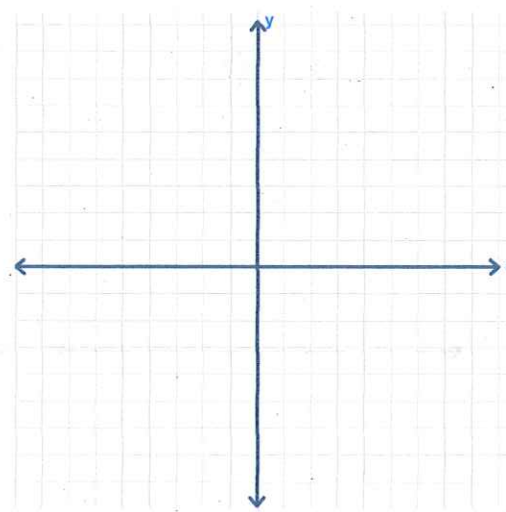
		Transformation rule:
1. Vertical shift *c* units *upward:*	$h(x)=f(x)+c$	$(x,y)\to(x,y+c)$
2. Vertical shift *c* units *downward:*	$h(x)=f(x)-c$	$(x,y)\to(x,y-c)$
3. Horizontal shift *c* units to the *right:*	$h(x)=f(x-c)$	$(x,y)\to(x+c,y)$
4. Horizontal shift *c* units to the *left:*	$h(x)=f(x+c)$	$(x,y)\to(x-c,y)$

Example:

Describe in words how the graph $y=(x+2)^2+3$ will be transformed from the graph of $f(x)=x^2$, then sketch the new graph



Try this: Describe, in words, how the graph $y=(x-4)^2+2$ will be transformed from the graph of $f(x)=x^2$, then sketch the graph



Reflecting Graphs

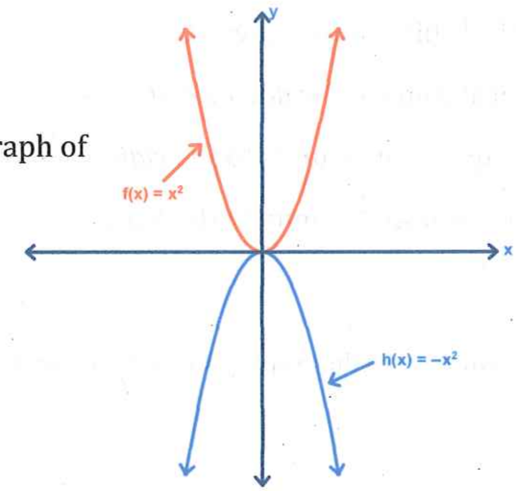
The second common type of transformation is a *reflection*.

If you consider the *x*-axis to be a mirror, the graph of

$$h(x) = -x^2$$

is the **reflection over the x-axis** (or mirror image) of the graph of

$$f(x) = x^2$$

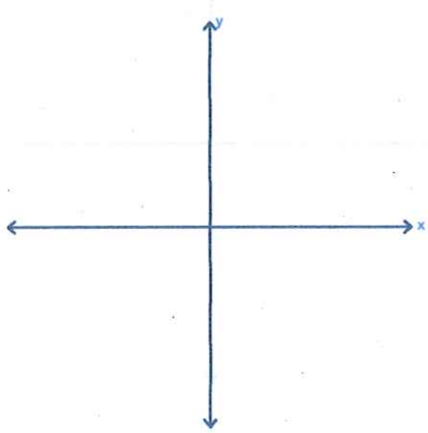


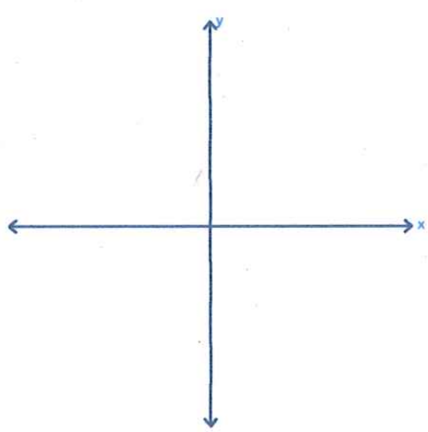
Reflections in the coordinate axes of the graph of $y = f(x)$ are represented as follows:

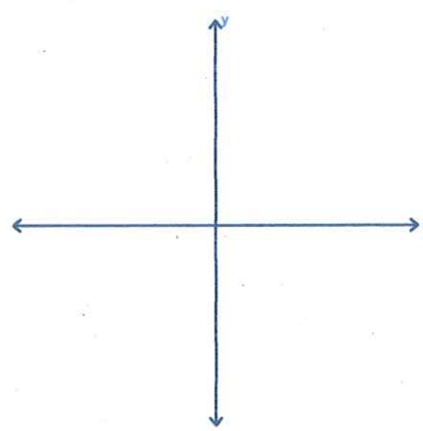
		Transformation rule:
1. Reflection in the *x*-axis:	$h(x) = -f(x)$	$(x,y) \to (x,-y)$
2. Reflection in the *y*-axis:	$h(x) = f(-x)$	$(x,y) \to (-x,y)$

Example:
Sketch the following graphs without using your calculator:

$y = -\sqrt{x+2}$ $\qquad$ $y = \sqrt{-x+2}$ $\qquad$ $y = \sqrt{-(x+2)}$







Nonrigid Transformations

The transformations we just discussed are *rigid* since the basic shape of the graph is unchanged....only the position of the graph is affected. *Nonrigid transformations* are those that cause a distortion or change in the shape of the original graph. We will be considering vertical and horizontal *stretches* and *shrinks*.

Vertical and Horizontal stretches and shrinking of the graph of $y=f(x)$ are represented as follows:

	Transformation rule:
1. $g(x)=c\bullet f(x)$ vertical stretch given that $c>1$	$(x,y)\to(x,cy)$
2. $g(x)=c\bullet f(x)$ vertical shrink or compress given that $0<c<1$	$(x,y)\to(x,cy)$
3. $g(x)=f(c\bullet x)$ horizontal shrink or compress if $c>1$	$(x,y)\to\left(\frac{x}{c},y\right)$
4. $g(x)=f(c\bullet x)$ horizontal stretch given that $0<c<1$	$(x,y)\to\left(\frac{x}{c},y\right)$

Example:
Describe the non-rigid change for the following function in comparison to the parent function:

$$y=\frac{1}{3}(x-2)^2$$

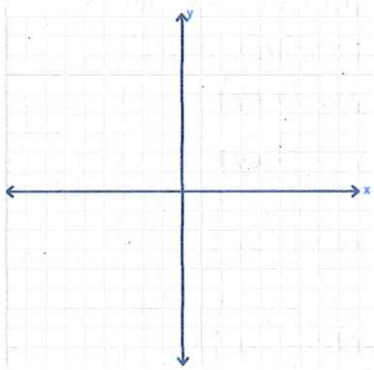
$$y=(4x)^2$$

$$y=\left(\frac{2x}{5}\right)^3$$

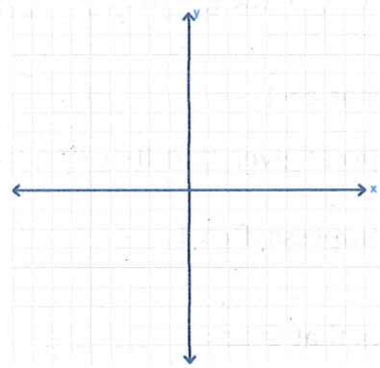
$$y=\frac{2}{5}\sqrt{x-3}$$

Example:
Sketch the following graphs without using your calculator:

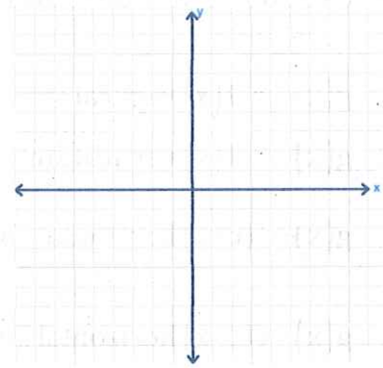
$y=(x+1)^2$



$y=2(x+1)^2$

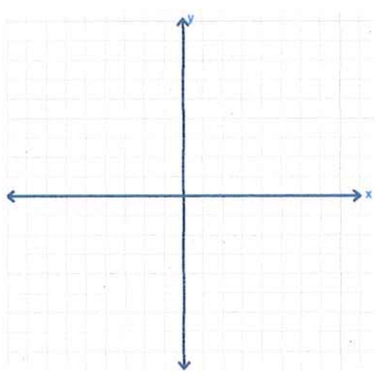


$y=\left(\frac{1}{2}(x+1)\right)^2$

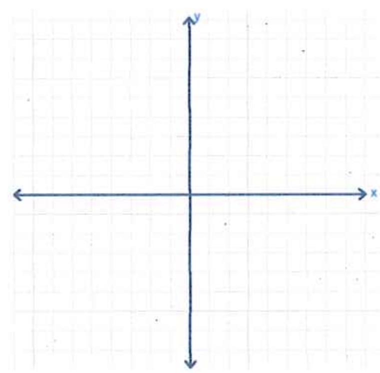


Try these:

$y=3(x+1)^2$



$y=(2(x+1))^2$



$y=\frac{1}{2}(x+1)^2$